

[Reviewed by Michael Berg, on 12/8/2011]

If I may start with a personal kvetch, I am at the moment the recipient of both a blessing and a curse. Specifically, a mathematical colleague and I currently comprise half of a working-group whose *raison d'être* is a new look at certain themes in quantum mechanics from a differential and algebraic topological perspective. It is a wonderful and interesting business, seeing that, for instance, sheaf cohomology (one of my favorite things) has become a big player in what we do. But this silver lining has a dark cloud: the other half of the group consists of a physicist and a rocket scientist keen on, and well versed in, physics *qua* physics, meaning that their handling of certain mathematical themes is often very, very different from the way we mathematicians do it — and this really is something of a curse, I think, given that communication gets dreadfully difficult from time to time. When physicists speak Mathematicalese it sounds an awful lot like Pig-Latin, at least to me (for what it's worth: my mathematical colleague is himself a convert from some species of science so he is better able to deal with all this), and I certainly need a dictionary.

Well, it looks like now there is one, *viz.* the book under review. The appearance of Jeevanjee's *An Introduction to Tensors and Group Theory for Physicists* is an example of excellent timing: I find in its six chapters at least three treatments of themes that arise all the time in my working-group and generally cause problems *vis à vis* mutual or intramural understanding. Specifically, chapter 3 concerns tensors, chapter 4 Lie theory, and chapter 5 representation theory. Tensors, or tensor products, are by themselves an abundant illustration of the bizarre acts physicists routinely perform on mathematical objects. The business of raising and lowering indices, often fitted into the mixed blessing of Einstein's summation convention, is found everywhere in modern physics, especially when relativity rears its head. I find it painfully ironic that two of my three favorite physicists, Einstein and Dirac, claim credit for trickery of this sort — recall Dirac's bra-kets. In point of fact, and to be fair, it is all quite safe and very clever, but it's still *unheimlich* to me, and I claim that I am not alone: to a large degree mathematicians and physicists are, as the phrase goes, separated by a common language.

And also by differing perspectives, of course. Consider, for instance, the following not-quite-opening sentences penned by Jeevanjee in the Preface:

*“My original motivation for the book was to demystify tensors ... The word tensor is ubiquitous in physics ... and yet tensors are rarely defined carefully ... [T]h definition usually has to do with transformation properties, making it difficult to get a feel of what these objects are ... [Aha!] ... In Part II ... I introduce group theory and its physical applications ... [and] [i]n Chap. 5 I introduce representation theory, which is a mathematical formalization of what we mean by the “transformation properties” of an object ...”*

Accordingly we can perhaps begin to delineate the problem: the physicists' notion of definition is not ours — they want to get a feeling of *what things are*, and they're after mathematical formulations of things they believe they already have a feeling for, while evidently trying to be as flexible as possible. This is legitimate, of course, but it's not quite our style.

This only increases the value of Jeevanjee's undertaking, particularly as he “aims ... to be simultaneously intuitive and rigorous ... [as well as] informal [and] ... precise.” If he succeeds he will have provided us with the dictionary I postulated above, and I will take it with me to my next working-group meeting.

Well, what does he do? Regarding tensors, on which the book is focused by the author's own admission, he takes the reader in a little over 80 pages from “A quick introduction” all the way to a discussion of how these objects perform in classical as well as quantum physics, and a treatment of symmetric as well as antisymmetric tensors. In the interim Jeevanjee hits the requisite material on dual spaces, for example, and he addresses the business of contraction of tensors on p. 56, going on quickly thereafter to resolving the identity in full-fledged Dirac notation. This illustrates the main feature of the book *qua* pedagogy: true to his word, Jeevanjee takes time and expends effort to teach the reader the “slang” physicists use in connection with these notions that, to us mathematicians, have such a different flavor (to wit: I first learnt about tensor products in terms of dividing out by bilinearity and, subsequently, the according universality property *vis à vis* bilinearity in mappings on a Cartesian product of two vector spaces. No doubt that is the rule rather than the exception for us mathematicians. But this approach is nowhere to be found in the book under review, and I guess that's consonant with how the physicists do things).

This complementarity of perspectives is carried over into, for example, Jeevanjee's treatment of representation theory: it reads very different from, e.g., Serre's gorgeous book, *Linear Representations of Finite Groups*. This is a telling fact, seeing that Serre set himself the task of addressing (at least in the first part of the book), of all things, quantum chemists! But no one doubts the huge value of Serre's book for mathematicians, so the fact that Jeevanjee's approach varies from Serre's is meaningful: it's all about how the physicists think about and use representations, and the book under review is laden with important examples. Lie groups and Lie algebras are prominently featured, as, of course, they should be.

So, the verdict is obvious, really. Jeevanjee's *An Introduction to Tensors and Group Theory for Physicists* is a valuable piece of work on several counts, including its express pedagogical service rendered to fledgling physicists and the fact that it does indeed give pure mathematicians a way to come to terms with what physicists are saying with the same words we use, but with an ostensibly different meaning: it's the same meat and potatoes, really, but the flavoring is all different.

Oh yes, one more thing. The book is very easy to read, very user-friendly, full of examples (as I already hinted) and exercises, and will do the job the author wants it to do with style. I am indeed going to use it myself, hopefully to great advantage, in my upcoming dealings with my working-group.