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★**An introduction to tensors and group theory for physicists.**

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The main objective of this textbook is to demystify the theory of tensors and their group-theoretical applications in physics by developing the fundamental facts step by step, and focusing only on some of the fundamental Lie groups like $\mathrm{SU}(2)$, $\mathrm{SO}(3)$ and $\mathrm{SO}(1,3)$. Rather than just presenting a full and detailed account of these groups, the author seeks to illustrate why such groups are so important and constitute the core of group-theoretical methods. The book is divided into two distinct parts, the first one (Chapters 1–3) dealing with linear algebra and tensors, the second focusing on group theory in physics (Chapters 4–6).

Chapter 1 explains the intention of the book and serves as an introduction. The second chapter is merely a review of results of linear algebra that are essential for the tensor and group theory applied to physical phenomena. In addition to sections covering the elementary notions of linear spaces, their properties and generic linear operators, two sections are devoted to the important case of linear spaces endowed with a Hermitian form, the latter constituting the prototype of the inner product used in physics. It is exactly at this point that it is convenient to introduce the Dirac notation and carefully explain its particularities. Further reasons for the convenience of this nomenclature will appear naturally during the discussion of linear representations of groups.

Chapter 3 introduces the notion of tensors using the algebraic formalism. That is, a tensor is considered as a multilinear form of a vector space isomorphic to the Cartesian product of r copies of a vector space V and s copies of its dual space V^*. This formal derivation of the concept illustrates clearly the classical notion of tensors given in most physics textbooks, namely as a set of scalars transforming according to a given set of rules. It turns out that, using this formalism, "defining" a tensor in the usual way is nothing more than giving the components of such a multilinear form once a basis of V (and hence of V^*) has been fixed, and helps to explain the transformation rules, which are those obtained from a change of reference. As obvious as the latter fact may appear, it actually constitutes one of the difficulties for beginners to really understand the notion of tensors and tensor fields. Once the basic properties of tensors have been introduced and physically motivated by means of well-chosen examples, the author considers the main algebraic operations for tensors. Among these, the tensor product is the most important operation considered, and will play a crucial role in the second part of the book. Another rather important technical notion studied in this part and that will appear later is the distinction between symmetric and skew-symmetric tensors. With these, the notions of boson and fermion (symmetrization postulate) can be introduced without direct reference to their underlying spin-statistics context. Some additional properties of these objects are postponed until Chapter 5, when the parity map is discussed.

The second part of the book deals with group theory, where the notions from the first part are applied by means of representation theory. Although the title of this part refers to (Lie) groups and algebras, and some definitions of rank-n groups are given, most of the attention is focused on $\mathrm{SO}(3)$, $\mathrm{SU}(2)$ and the proper orthochronous Lorentz group. This choice is far from being coincidental, for these groups are in fact the first ones encountered in physical applications (up to $\mathrm{U}(1)$), and their structure and representation theory are the keystone for higher-dimensional simple Lie groups. Chapter 4 introduces these groups and some general properties of Lie groups, such as homomorphisms, subgroups, the associated Lie algebra as tangent space at the identity and the exponential map, providing a (local) reconstruction of the group. Some additional properties, like the Killing form, are briefly discussed as problems at the end of the chapter.

Chapter 5 presents the main facts concerning representations of Lie groups and algebras. Starting with the regular and adjoint representations, such important notions as unitarity, left-handed (right-handed) spinor representations, pseudo-vector representations, etc., are introduced and illustrated with the rank-one simple Lie groups. For completeness in the presentation, some material corresponding to the discrete symmetries, like the parity map, is also mentioned. The author introduces the tensor product as a fundamental technique to generate representations from given ones, pointing out the important distinctions between multiplets arising from symmetric or skew-symmetric tensor products. At this point the dual representations and the self-adjoint representations are discussed, leading naturally to the characterization problem. The complete reducibility of representations and the range of validity of this property are discussed, prior to a detailed analysis of the representations of rank-one simple Lie groups and the Lorentz group.

The sixth and last chapter is physically the most relevant, as it deals with the fundamental Wigner-Eckart theorem and its applications to quantum mechanics. This is both the most important point in the study and a quite delicate question. However, the author masters the presentation without invoking too much formalism, and nicely clarifies the notions of tensor operators or spherical tensors, traditionally two concepts that are hard for beginners to digest. Using the material previously considered on linear spaces and representations, the author introduces the selection rules as orthogonality relations. He then shows that the Wigner-Eckart theorem constitutes a complement to the angular momentum selection rule. The connection with Clebsch-Gordan coefficients (the usual statement of this theorem) is then straightforward. The chapter ends with a brief section devoted to the gamma matrices.

Two appendices contain the basic material concerning the properties of complexified Lie algebras and the representation theory of A_1 seen as a real Lie algebra.

Although this book presents only introductory material concerning Lie groups and algebras in physics, it covers many aspects and results that in formal texts are usually assumed to be known implicitly by readers or are merely mentioned in appendices and hence constitute a non-negligible difficulty for the freshman encountering the theory for the first time. Its goal is not to help students to deal with and manipulate tensor objects, but rather to understand what is actually going on. Once this understanding is established, many of the techniques implicitly assumed in classical or quantum mechanics and other areas like general relativity will become much clearer and more transparent. Each chapter moreover presents a collection of problems of varying difficulty that should help to consolidate the material studied, as well as to illustrate more advanced applications of the techniques that are developed. Both the theoretical and the practical parts are well balanced and provide a solid background for students, helping them to understand the more advanced literature on the subject without formal difficulties.

In conclusion, this book not only fills a considerable pedagogical gap in the physical and mathematical literature, but also shows to what extent the material arises naturally within any consistent model of natural phenomena. There is little doubt that this textbook will become a canonical reference for students in the coming years.

Reviewed by *Rutwig Campoamor-Stursberg*