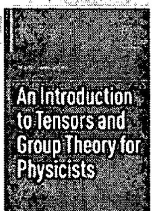
An Introduction to Tensors and Group Theory for Physicists

Nadir Jeevanjee
Springer, New York, 2011.
$59.95 (242 pp.).
ISBN 978-0-8176-4714-8



Physics and mathematics depend on each other, though their languages and concerns often diverge. More and more, abstract mathematics has come to figure centrally in the working vocabulary of physicists. In particular, tensors and groups have become familiar tools of theoretical physics, yet their abstractness tends to preclude physical understanding. Furthermore, contemporary mathematicians approach those topics in ways unfamiliar to most physicists. Merely repeating the old definitions and proofs is not sufficient to clarify their deeper conceptual significance.

With *An Introduction to Tensors and Group Theory for Physicists*, physicist Nadir Jeevanjee has produced a masterly book that will help other physicists understand those subjects as mathematicians understand them. In physics, tensors are often considered in terms of their coordinate representations, which seem well suited to physical systems. Modern mathematics treats them as more general objects, best understood apart from particular coordinates. The most abstract formulations, however mathematically rigorous, tend to strike physicists as too distant from their own concerns. Advanced undergraduates and beginning graduate students, to whom this book is directed, particularly struggle to take the first steps with those challenging approaches.

From the first pages, Jeevanjee shows amazing skill in finding fresh, compelling words to bring forward the insight that animates the modern mathematical view. In contrast to the usual description of a baffling beast bristling with indices, Jeevanjee describes how, as he puts it, tensors eat vectors and spit out numbers. He combines vivid use of language with coherent expositions of the detailed equations and expressions. Above all, with compelling force and clarity, he provides many carefully worked-out examples and well-chosen specific problems.

In the five-page introduction, Jeevanjee plunges the reader into the heart of the matter by giving a clear overview of the physical and mathematical views of tensors. Ironically, the definition of tensors in terms of transformation properties, though most familiar to physicists, is more abstract than needed. Jeevanjee shows how modern mathematics derives those properties from the simpler, more fundamental role of tensors to produce numbers from vectors. He then explains vector spaces and their duals, fleshed out by detailed applications to familiar examples from classical mechanics and basic quantum theory. In just 80 pages, *An Introduction to Tensors and Group Theory for Physicists* builds a rounded view that includes tensors' modern mathematical presentation and their physical applications, yet without hurrying and without overtaxing the reader's stamina.

The remaining 150 pages accomplish a similar feat for group theory, explaining it and applying it to examples that bring the fundamental mathematical principles to life. Jeevanjee's clear and forceful writing presents familiar cases with a freshness that will draw in and reassure even a fearful student. He does not stint the technical details, which are nicely embedded in the text so that they connect smoothly with the larger conceptual exposition.

Jeevanjee's treatment clarifies Lie groups and algebras, as well as tensor products, whose distinctions and precise meanings so often perplex students. His preceding presentation of tensors gives a helpful foundation on which he builds group-theoretic concepts. Jeevanjee outlines the structure of the orthogonal and Lorentz groups, the special unitary groups $SU(n)$, and complex groups based on linear groups such as $SL(n)$. He then moves from a general overview to specific cases, especially drawing on $SU(2)$, so important for spin in quantum theory. His presentation includes excellent explanations of covering groups; in the process, he gently introduces Lie brackets (from Poisson brackets) and the mathematical language of homomorphisms. The final chapters treat basic representation theory, especially for spinors; they introduce the intertwiner, a particularly helpful invention of modern mathematics; and they end with the Wigner–Eckart theorem and other applications.

Topics in *An Introduction to Tensors and Group Theory for Physicists* nicely complement those of Pierre Ramond's excellent introductory book, *Group Theory: A Physicist's Survey* (reviewed in PHYSICS TODAY, June 2011, page 53). Unlike Jeevanjee's text, that book, oriented toward the standard model of particle physics, goes into the detailed representation theory of $SU(3)$. Jeevanjee instead concentrates on connecting tensors and group theory with modern mathematics, while also showing their physical applications.

An Introduction to Tensors and Group Theory for Physicists, written during Jeevanjee's graduate studies at the University of California, Berkeley, is a masterpiece of exposition and explanation that would win credit for even a seasoned author. One can only hope that, after this prodigious first book, he will write many more.

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